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Mean field model for ferromagnetic nanowire arrays based on a mechanical analogy

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Abstract
Nanowire arrays have typical distributions of coercive and interaction fields, as revealed in experiments by means of the first-order reversal curve (FORC) method. In an axial applied field, each nanowire is subjected to a state dependent interaction field created by all wires from the array, which strongly influences the switching fields. A mean field model based on Preisach–Krasnosel’skii–Pokrovskii (PKP) hysterons is able to explain the interplay between interactions and critical fields in nanowire arrays and the subtle way in which switching fields are emphasized in FORC diagrams.

1. Introduction

Systems of ordered ferromagnetic nanowires have been extensively studied in recent years due to their potential in the field of technological applications such as magnetic sensors, advanced data storage and as elements in microwave devices [1–3]. Tailoring their magnetic properties by means of geometrical characteristics, such as the nanowire dimensions and the array periodicity, represents a huge advantage for specific practical applications based on nanowire arrays.

Two important issues are involved in the magnetic response of a nanowire array to the external field, especially when it is applied parallel to the nanowire’s axis. On one hand, the shape anisotropy, basically due to the cylindrical form with high aspect ratio, tends to maintain the magnetization direction along the wire axis. In modern fabrication processes, especially based on anodized alumina templates, a better control of the wire diameters can be obtained and consequently intrinsic coercive fields of the magnetic wires have almost the same value, which is equivalent to a very small dispersion of the coercive field distribution. On the other hand, interwire magnetostatic interactions significantly affect magnetic properties, mainly magnetization switching of the nanowires. Considering an axial applied field, the interaction field appears as a collective effect of the fictitious magnetic charges located at the ends of the nanowires. While the length of the wires is much higher compared with their diameter, it is natural to consider long-range interaction fields that generate a mean interaction field over the whole nanowire array [4].

A powerful experimental tool used to evaluate coercive fields and interactions in any ferromagnetic system is the first-order reversal curves (FORC) diagram method, which provides a distribution of coercive and interaction fields, called FORC distribution (usually represented as a contour plot diagram). Initially developed by Mayergoyz [5, 6] as an identification procedure for the classical Preisach model (CPM) [7], the FORC protocol is now extensively used in many laboratories in magnetic characterization of ferromagnetic samples even when they are not necessarily CPM systems complying with the representation theorem [5].

In Preisach modeling, mean field interactions dependent on the magnetic state of the sample were described by Della Torre [8] as a version of the initial model, called the moving Preisach model (MPM). Pike and co-workers have extended the applicability of the FORC method to any hysteretic system [9], and the link between the mean field interactions of magnetizing/demagnetizing type and FORC diagram features was first evidenced by Stancu et al [10].
FORC diagrams have a specific shape for nanowire arrays, constantly appearing as a high extension on the interaction field axis with a narrow dispersion of coercivities, and in addition an unexpected supplemental distribution of higher coercive fields with weak interactions is always present [11–19]. The origin of this supplemental distribution is still a controversial issue, and some different assumptions were made in order to explain their provenance [11, 15, 19]. In this paper, after a presentation of the FORC diagram method, we propose a mean field model based on Preisach–Krasnosel’skii–Pokrovskii (PKP) hysterons [20–22] that can accurately generate typical FORC distributions for nanowire arrays as they are obtained in experiments. While the supplemental coercive field distribution exists in all FORC diagrams of the nanowire arrays, it is related to the ‘reversal-field memory’ effect [23] describing specific switching events of the wires [19], and is easily evidenced in our proposed model.

2. FORC diagram method

In the classical Preisach model, each particle is characterized by a rectangular hysteresis loop with switching fields \( H_a, H_b \), as in figure 1. Preisach distribution is the distribution of the hysteresons, more specific of the switching fields, and it is represented in the Preisach plane, using the coercive field axis \( H_c = (H_a - H_b)/2 \) and the interaction field axis \( H_i = -H_u = -(H_a + H_b)/2 \), where \( H_u \) is the shift of the hysteresis loop and is usually called the ‘interaction field’.

A FORC measurement starts from the descending branch of the major hysteresis loop (MHL) at a reversal field \( H_r \). The sample’s magnetic moment on a FORC for a given reversal field is a function of two variables, \( m_{\text{FORC}}(H, H_r) \), where \( H \) is the applied field during the measurement, increased in the experiment until the saturation state is obtained again. The FORC function is proportional to the second-order mixed derivative of the magnetic moment measured for a set of FORCs and it is given by:

\[
\rho_{\text{FORC}}(H, H_r) = -\frac{1}{2} \frac{\partial^2 m_{\text{FORC}}(H, H_r)}{\partial H \partial H_r}.
\]

The FORC diagram is the contour plot of the corresponding distribution and it is usually represented in \((H_c, H_u)\) coordinates to directly obtain coercivities and interaction fields of the particles within the magnetic system. In the classical Preisach model, the FORC function represents the statistical distribution of Preisach hysterons described by the switching fields \( H \equiv H_a \) and \( H_i \equiv H_b \).

Experiments [24] and micromagnetic simulations [25] have proved that one isolated nanowire has a rectangular and symmetrical hysteresis loop perfectly represented by a non-interacting Preisach hysteron. In order to describe ferromagnetic nanowire arrays by the instrumentality of the FORC diagrams, we define first an ideal non-interacting nanowire array as an infinite and perfectly ordered grid with identical and parallel wires with their axes perpendicular to a plane. Having the same coercive field, all wires switch simultaneously at the same value of the applied external field, so the FORC distribution for such an array is one Dirac peak located on the horizontal coercivity axis (see figure 2(a)). Consequently, one single symmetrical Preisach hysteron can describe the behavior of the mentioned ideal non-interacting nanowire array, as well as of one isolated wire.

![Figure 1. Preisach hysteron with a critical field, \( H_c \), and an interaction field, \( H_u \).](image)

![Figure 2. Modeled FORC diagrams for: (a) non-interacting Preisach hysterons, (b) a singular distribution of Preisach hysterons, or a population of non-interacting Preisach hysterons in a negative interaction mean field.](image)
Let consider now the same ideal nanowire array but with inter-wire interactions because of the physical closeness of the nanowires (a small lattice constant). The FORC diagram presented in figure 2(b) can represent a uniform distribution of interaction fields of a population of Preisach hysterons. In the same time, starting from a set of non-interacting Preisach hysterons and adding a negative moving term, we obtain the same FORC diagram where \(\Delta H_u\) is the maximum value of the interaction mean field, i.e. in the saturated state. Nanowire arrays are characterized by statistical interaction fields with a narrow dispersion, on one hand due to a high degree of order, and on the other hand due to the long-range interaction fields. So a wire is experiencing the field created not only by a few neighboring wires, but by all the other wires from the network [4]. For these reasons, mean field interactions are representative for nanowire arrays, and experimentally observed FORC diagrams may be accurately interpreted in a mean field approach.

In the Preisach community, it is common knowledge that in an axial applied field nanowire arrays generate negative mean field \((\alpha m, \text{where } \alpha < 0 \text{ is the moving constant and } m \text{ is the total magnetic moment of the array}), \) usually called a ‘demagnetizing-type mean field’. Obviously \(\alpha m < 0\) when \(m > 0\) and \(\alpha m > 0\) when \(m < 0\) so, as a global effect, the mean field remains always anti-parallel to the total magnetic moment of the array. When we analyze a switching event of a selected nanowire, it is important to understand the effect of this mean field on that nanowire. Precisely, as a local effect, it is important to know if the mean field is parallel or anti-parallel to the nanowire magnetic moment, and not with respect to the total magnetic moment of the array. For example, on the superior half of the descending branch of MHL \((m > 0)\), unswitched nanowires are subjected to demagnetizing mean fields, anti-parallel to their magnetic moments. On the inferior half of the descending branch of MHL \((m < 0)\), unswitched nanowires are subjected to magnetizing mean fields, in the same direction as their magnetic moments, even if, in the same time, the mean field is anti-parallel to the total magnetic moment of the array.

3. PKP hysterons

In Preisach modeling, a hysterons is usually associated with a particle from a system and it is a basic element for a hysteresis operator. To provide a simple description of a nanowire array, we introduce first a single hysterons, starting from a mechanical analogy, that can be associated with the entire interacting nanowire array, namely with one interaction mean field of demagnetizing-type.

A model for rigid plasticity can be built as a parallel connection of a perfect elastic element and a rigid perfectly plastic one [20, 21]. This is a ‘play hysterons’ and it is also used in Preisach-type models in ferromagnetism [26, 27]. Both elements are respectively represented in figure 3 by a spring with linear dependence of the deformation \((\varepsilon)\) on the stress \((\sigma_\varepsilon = -ae)\) and a heavy body sliding on a horizontal surface with dry friction \((\sigma_c)\). Limiting the displacement of the plastic element at extreme values \(\pm \varepsilon_s\), the play hysterons becomes a saturated play hysterons which will be called a negative PKP hysterons [18] for a realistic elastic constant, \(\alpha < 0\).

Starting from \(\sigma = \varepsilon = 0\), for a quasi-static increase of the applied stress, the system behaves rigidly until the critical value \(\varepsilon_c\) is reached. For \(\sigma > \sigma_c\) the deformation becomes proportional to \(\sigma - \varepsilon_c\) until the maximum value \(\varepsilon_s\) is reached and for \(\sigma > \sigma_c + \varepsilon_s\) the displacement cannot increase due to the limitation. If in any state the applied stress is decreased, the displacement in the opposite direction occurs when \(\sigma + \varepsilon_c\) attains the opposite critical value \(-\varepsilon_c\). In summary, this ideal mechanical system leads to a parallelogram-shape MHL inside of which the superior order reversal curves can be completed only in the horizontal direction.

It is easy to remark that in the absence of the elastic element, the PKP hysterons becomes a rectangular symmetrical hysterons, as in figure 1 for \(H_u = 0\). The elastic stress has a feedback role generating a linear dependence of the input \((\sigma)\) on the output \((\varepsilon)\), which causes the clockwise rotation of the MHL. This is, in fact, the essence of the moving Preisach model [8] in the case of the negative mean field, so \(ae\) represents the moving term and \(\alpha < 0\) signifies the moving constant.

Now, the correspondence is obvious between the mechanical parameters from figure 3 and the magnetic ones we see in figure 4(a), in which the magnetic moment, \(m\), is normalized for convenience. For an ideal interacting nanowire array, the interaction mean field is \(H_i = \alpha m = -H_u\), with \(\alpha < 0\). A negative PKP hysterons is parameterized by the coercive field, \(H_c\), and the interaction field in the saturated state, \(H_{is} = \alpha = -H_us\). It describes the mean field interactions in the ideal infinite nanowire array, in which each switching of a given nanowire, from ‘up’ to ‘down’ and back from ‘down’ to ‘up’ takes place in the same interaction field, so at the same magnetic moment of the nanowire array:

\[
H_\beta = -H_c - \alpha m, \quad H_\alpha = H_c - \alpha m.
\]  

Using again the mechanical analogy, we describe a positive mean field that can be associated with some
nanowires from the array. Let us consider that the elastic element from figure 3 has an unrealistic positive constant, $a > 0$, that leads to an unrealistic MHL with instabilities. The hysteresis loop is also a parallelogram but oriented counterclockwise, as we present in figure 4(b) for magnetic parameters. In order to make sense for this positive moving effect, we transform the loop in a rectangular loop with an apparent higher coercivity and we call it a positive PKP hysteron. A positive PKP hysteron is characterized by the same parameters as a negative PKP hysteron, $H_c$ and $H_{ls} = \alpha = -H_{us}$, but with $\alpha > 0$, which leads in fact to a higher apparent coercive field:

$$H_{apparent}^c = H_c + H_{ls}. \quad (3)$$

It is interesting to observe that the switching fields are symmetrical and the transformed hysteron appears as a Preisach hysteron without interaction field. In fact, if for a given nanowire the interaction fields in switching moments (from ‘up’ to ‘down’ when the array magnetization is $m_\uparrow$ and from ‘down’ to ‘up’ when the array magnetization is $m_\downarrow$) are equal and opposed, $\alpha m_\downarrow = -\alpha m_\uparrow < 0$, the switching fields are equal and opposed as well:

$$H_\beta = -H_c + \alpha m_\downarrow, \quad H_\alpha = H_c + \alpha m_\uparrow. \quad (4)$$

### 4. Switching events evidenced in the FORC diagram

In figure 5 are presented FORCs for a pair of PKP hysteron, a positive and a negative one, both of them having the same coercive field $H_c = 150$ Oe and the interaction fields $H_{ls} = \pm 100$ Oe, which leads, using (3), to $H_{apparent}^c = 250$ Oe in the case of the positive PKP hysteron.

Considering an infinite nanowire array, the negative PKP hysteron describes nanowires that switch ‘down’ on the MHL and switch ‘up’ on the FORC at the same magnetic moment of the sample and consequently in the same interaction mean field (see (2)). As an example, in figure 5 we emphasize one Preisach hysteron for a nanowire that switches in $B_\downarrow$ and in $B_\uparrow$ (when the magnetic moment of the sample is near $-1$), and this is represented in the FORC diagram from figure 6 in B. All nanowires that switch in this fashion are described by the negative PKP hysteron which corresponds to the entire distribution AB from figure 6, with interaction fields between $\pm 100$ Oe. Following the effect of the mean field on a selected nanowire when it switches, we observe different effects in the two switching moments, as can be seen in figure 5: one switching event takes place in a demagnetizing mean field and the other one in a magnetizing mean field (e.g. the pairs of switchings $A_\downarrow$–$C_\uparrow$ and $B_\downarrow$–$B_\uparrow$). Consequently, the Preisach hysteron associated with the nanowires is shifted along the field axis, being asymmetrical. While experimental FORC diagrams evidenced a prominent distribution extended along the $H_u$ axis with a small dispersion of the coercive fields [11–19], we can conclude that most switching events can be described by the negative PKP hysteron.

All experimental FORC diagrams present an additional distribution of higher coercive fields without interactions. While this distribution exists even for arrays with a very small dispersion of the geometrical characteristics of the nanowires,
it cannot be attributed to an intrinsic distribution of coercive fields that is usually a narrow one. However, as we have seen in figure 4(b), switching events described by positive PKP hysterons lead to higher apparent coercive fields. In figure 5 we have illustrated a possible switching of a nanowire, from ‘up’ to ‘down’ in B↓ when the magnetic moment of the array is near −1 and from ‘down’ to ‘up’ in C↑ when the magnetic moment is near +1. In both switching moments, the nanowire is subjected to a mean field with magnetizing effect because between A↑ and B↓ almost all the other wires from the array have switched ‘down’, and they have switched ‘up’ between B↑ and C↑. This specific type of switching is described by a positive PKP hysteron, for which the effect of the interactions is actually the increase of the Preisach hysteron coercivity. Using (4) we obtain $H_a = -H_{B\downarrow}$, so we have a symmetrical Preisach hysteron corresponding to the point C in the FORC diagram in figure 6.

The nanowires from a real array are not all identical and perfectly parallel, so a distribution of intrinsic coercive fields exists, even if it is a narrow one. On the descending MHL, the nanowires with higher intrinsic coercive fields switch ‘down’ when the applied field is lower than $-H_c$, so the mean field has a magnetizing effect on these nanowires (see figure 5). If on the FORC that starts after such a nanowire has switched ‘down’, nanowires with lower coercivities switch first ‘up’, and the higher coercive nanowires will switch ‘up’ among the last when the applied field is higher than $+H_c$, also in a mean field with magnetizing effect (see again figure 5). So, because there is a distribution of coercive fields, the nanowires with higher intrinsic coercive fields will switch in this fashion, and switchings are evidenced in FORC diagram at higher coercive fields.

In figure 6 we present, as an example, a FORC diagram with a typical shape for nanowire arrays, obtained with one negative PKP hysteron having $H_{B\downarrow} = -100$ Oe (attributed to 85% of total switchings) and 50 positive PKP hysterons uniformly distributed with $H_{B\uparrow}$ between 0 and 100 Oe (associated with 15% of the total switchings), considering that all hysterons have the same intrinsic coercive field, $H_c = 150$ Oe. While in reality the critical fields of the nanowires with which we have associated positive PKP hysterons are only slightly higher than those of nanowires described by the negative PKP hysteron, our consideration does not significantly affect the FORC diagram. Symmetrical Preisach hysterons can be obtained only for nanowires which reverse down just before the FORC starts from the negative magnetic moment of the array, $m_{B\downarrow}$, and reverse up at the same absolute value of the magnetic moment but positive $m_{B\uparrow} = -m_{B\downarrow}$, meaning that in the FORC diagram the supplemental distribution can be extended on the segment $C_0\downarrow C_0\uparrow$. One can observe that $C_0\downarrow C_0\uparrow B$, which is consistent with all experimental FORC diagrams [11–19].

The FORC diagram presented in figure 6 is obtained using a single PKP negative hysteron, which is consistent with the assumption that the array is infinite and contains identical and perfectly ordered nanowires. Real nanowire arrays are of course finite, but a rather small set of negative PKP hysterons (usually 5–10 hysterons) associated with the array are sufficient to take into account the variation of the interparticle interaction fields from the center to the periphery of the array, evidenced in experimental FORC diagrams [11–19]. In [4] we have proposed an identification procedure for any polygonal-shaped array divided into the regions containing nanowires with roughly the same interaction mean field in the saturated state of the array. Also, the saturated magnetic moments of the negative PKP hysterons can be setup using the weights of the regions. A distribution of coercive fields can be introduced by replacing each negative PKP hysteron by a set of the same type of hysterons (for example 50 hysterons) but with distributed coercivities with a small dispersion, in order to obtain the best fit to the experimental data.

Note that a negative PKP hysteron is associated to a negative mean field, while a positive PKP hysteron, transformed in a symmetrical Preisach hysteron, is associated to a nanowire. The distribution of high coercive fields observed in FORC diagrams may be modeled with classical Preisach hysterons with correspondingly high coercivities and zero interaction fields, but without any physical explanation. The positive PKP hysteron is proposed specifically to emphasize the subtle effect of a global negative mean field on switching events that take place when the nanowires feel the mean field with a magnetizing effect. The rate of these interesting events from the total switchings can be found from the experimental FORC diagram.

Higher coercive field distributions have been also observed in FORC diagrams obtained in the Edwards–Anderson Ising spin glass model [27] and in those of the patterned media [28]. The effect was named ‘reversal-field memory’ and a physical explanation was offered taking into account particles that switch last in their local neighborhood. In nanowire arrays, each nanowire is subjected to an interaction field created by a large number of neighbors. A distribution of intrinsic coercive fields exists, even if it is a narrow one, and the interplay between interactions and coercivities is responsible for symmetrical switching events reflected in higher coercive fields in FORC diagrams.
5. Conclusion

Starting from a mechanical approach, we have proposed a simple model based on PKP hysterons representing negative or positive mean fields in which the reversals take place. Using a small number of PKP hysterons we have provided a FORC diagram with specific distributions, which represents the basic diagram for ferromagnetic nanowire arrays. Magnetization reversals in nanowires were correlated with the mean field evolution reflected in the FORC diagram. Generally, a negative mean field characterizes a nanowire array, but switching events of the nanowires with slightly higher intrinsic coercive fields are governed by a positive mean field, which leads to higher apparent coercive fields experimentally observed for nanowire arrays.

An advantage of the proposed model is the fact that a small set of PKP hysterons can be used for better fitting of the experimental FORC diagrams in order to obtain information about switching events. A refined analysis of the evolution of the interaction fields, based on a physical approach, provides a better insight into the specific switching phenomena in nanowire arrays. Tailoring the apparent coercive fields, by means of the interaction fields, could be useful in applications requiring high coercivities.

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